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06MAT11

First Semester B.E. Degree Examination, June/July 2013

Engineering Mathematics - I

Time: 3 hrs. Max. Marks:100

Note: 1. ***Answer any FIVE full questions, choosing at least two from each part.***
2. ***Answer all objective type questions only in OMR sheet page 5 of the answer booklet.***
3. ***Answer to objective type questions on sheets other than OMR will not be valued.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

PART – A

1 a. Choose the correct answers for the following : **(04 Marks)**

i) If ϕ be the angle between the tangent and radius vector at any point on the curve $r = f(\theta)$ then $\sin\phi$ equal to _______.

A) $\frac{dr}{ds}$ B) $r\frac{d\theta}{ds}$ C) $r\frac{d\theta}{dr}$ D) $\frac{ds}{d\theta}$

ii) The n^{th} derivative of sin x is ________.

A) sin x B) $\sin^n x$ C) sin nx D) $\sin\left(x+\frac{n\pi}{2}\right)$

iii) For the polar curve $r = 2\sin\theta$, the value of $\tan\phi$ is ______.

A) 0 B) $-\tan\theta$ C) $\tan\theta$ D) $-\frac{1}{2}\cos\theta$

iv) The value of $\frac{d^{n+1}}{dx^{n+1}}[x^n]$ is ________.

A) n! B) (n + 1)! C) (n – 1)! D) 0

b. Find the nth derivative of $\frac{x^2-4x+1}{x^3+2x^2-x-2}$. **(04 Marks)**

c. If $x = \sin t$, $y = \cos pt$, show that $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-(n^2-p^2)y_n = 0$. **(06 Marks)**

d. Show that the curves $r^n = a^n\cos n\theta$; $r^n = b^n\sin n\theta$ cut each other orthogonally. **(06 Marks)**

2 a. Choose the correct answers for the following : **(04 Marks)**

i) If $u = x^y$ the $\frac{\partial u}{\partial x}$ is ______.

A) 0 B) $y\,x^{y-1}$ C) $x^y \log x$ D) $y^x \log y$

ii) If $x = r\cos\theta$, $y = r\sin\theta$, then $\frac{\partial(x,y)}{\partial(r,\theta)}$ is equal to ______.

A) 1 B) r C) $\frac{1}{r}$ D) $-\frac{1}{r^2}$

iii) If $z = \sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$ then $x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}$ is ______.

A) z B) 2z C) tan z D) sin z

iv) If $u = \log\left(\frac{x^2}{y}\right)$ then $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}$ is _______.

A) 2u B) u C) 0 D) 1

2 b. If $u = x + y + z,\ v = y + z,\ w = z$ then evaluate $\dfrac{\partial(u,v,w)}{\partial(x,y,z)}$. **(04 Marks)**

c. If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, show that $\left(\dfrac{\partial}{\partial x}+\dfrac{\partial}{\partial y}+\dfrac{\partial}{\partial z}\right)^2 u = \dfrac{-9}{(x+y+z)^2}$. **(06 Marks)**

d. For an elastic material the Young's modulus E, the rigidity modulus μ and the Poisson's ratio, σ are connected by the relation $E = 2\mu(1 + \sigma)$. Error of 1% and -5% occur while measuring μ and σ respectively. If these errors compensate and yields the correct value of E, find the value of σ. **(06 Marks)**

3 a. Choose the correct answers for the following : **(04 Marks)**

i) The value of $\int_0^{\pi/2} \sin^6 x\,dx$ is _______.

A) $\dfrac{15\pi}{32}$ B) $\dfrac{5\pi}{32}$ C) $\dfrac{\pi}{2}$ D) $\dfrac{3\pi}{20}$

ii) The value of $\int_0^{\pi/2} \sin^2\theta.\cos^4\theta\,d\theta$ is _____.

A) $\dfrac{2}{5}$ B) $\dfrac{2\pi}{15}$ C) $\dfrac{2}{15}$ D) $\dfrac{\pi}{32}$

iii) If $I_n = \int_0^{\pi/4} \tan^n\theta\,d\theta$ then which of the following is true

A) $n[I_{n+1} + I_{n-1}] = 1$ B) $I_{n+1} + I_{n-1} = 1$ C) $n[I_{n+1} - I_{n-1}] = 1$ D) $I_{n+1} + I_n = 1$

iv) The curve $x^3 + y^3 = 30xy$ is symmetrical about the line

A) x – axis B) y – axis C) $y = x$ D) $x + y + 2 = 0$

b. Using reduction formula, find the value of $\int_0^1 x^{3/2}(1-x)^{3/2}dx$. **(04 Marks)**

c. Trace the curve $x^{2/3} + y^{2/3} = a^{2/3}$. **(06 Marks)**

d. Obtain the reduction formula for $I_n = \int_0^{\pi/2} \sin^n x\,dx$ where n is a positive integer. **(06 Marks)**

4 a. Choose the correct answers for the following : **(04 Marks)**

i) For the polar curve $r = f(\theta)$ the value of $\dfrac{ds}{d\theta}$ is

A) $\left[1+\left(\dfrac{dr}{d\theta}\right)^2\right]^{1/2}$ B) $\left[r^2+\left(\dfrac{dr}{d\theta}\right)^2\right]^{1/2}$ C) $\left[r^2+\left(\dfrac{d\theta}{dr}\right)^2\right]^{1/2}$ D) $r\dfrac{d\theta}{dr}$

ii) For the Cartesian curve $y = f(x)$ the value of ds/dx is equal to

A) $\sqrt{1+\left(\dfrac{dy}{dx}\right)^2}$ B) $\sqrt{1+\left(\dfrac{dx}{dy}\right)^2}$ C) $\sqrt{(dx)^2+(dy)^2}$ D) $\sqrt{1+\dfrac{d^2y}{dx^2}}$

iii) Area bounded by the curve $y = f(x)$, the x-axis and the ordinates $x = a$, $x = b$ is

A) $\int_a^b y\,dx$ B) $\int_a^b x\,dy$ C) $\int_b^a xy\,dx$ D) $\int_a^b (x+y)dx$

4 a. iv) The volume of the solid generated by the revolution about the x-axis, of the area bounded by the curve y = f(x), the X-axis and the ordinates x = a; x = b is ______.

A) $\int_a^b \pi x^2\,dy$ B) $\int_a^b \pi y^2\,dx$ C) $\int_0^a (x^2+y^2)\,dx$ D) $\int_a^b xy\,dx$

b. Find the perimeter of the Cardioid $r = a(1 + \cos\theta)$. **(04 Marks)**

c. Find the area bounded by the Cissoid $y^2(2a - x) = x^3$ and its asymptote. **(06 Marks)**

d. Evaluate $\int_0^\infty \frac{e^{-x}}{x}(1-e^{-ax})\,dx$ $(a > -1)$ by using Leibnitz's rule. **(06 Marks)**

PART – B

5 a. Choose the correct answers for the following : **(04 Marks)**

i) The solution of the differential equation $\frac{dy}{dx} + \frac{y}{x} = 0$ is _____.

A) xy = k B) x + y = k C) $x^2 = yk$ D) x/y = k

ii) The integrating factor (IF) of the differential equation $\frac{dy}{dx} + y\cot x = 4x\,\text{cosec}\,x$ is

A) cot x B) cos x C) sin x D) cosec x

iii) If the differential equation of the form M(x, y)dx + N(x, y)dy = 0 is exact then

A) $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y}$ B) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ C) $\frac{\partial M}{\partial y} = -\frac{\partial N}{\partial x}$ D) $\frac{1}{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}} = \varphi(x)$

iv) If $f\left(r, \theta, \frac{dr}{d\theta}\right) = 0$ be the differential equation of the family of curves f(r, θ, c) = 0 then the differential equation of orthogonal trajectories is

A) $f\left(r, \theta, -\frac{d\theta}{dr}\right) = 0$ B) $f\left(r, \theta, -r\frac{d\theta}{dr}\right) = 0$ C) $f\left(r, \theta, -r^2\frac{d\theta}{dr}\right) = 0$ D) $f\left(r, \theta, -r^2\frac{dr}{d\theta}\right) = 0$

b. Solve $e^y\left(\frac{dy}{dx} + 1\right) = e^x$. **(04 Marks)**

c. Solve (x + 2y) (dx – dy) = dx + dy. **(06 Marks)**

d. When a switch is closed in a circuit containing a battery E, a resistance R and an inductance L, the current i builds up at a rate given by $L\frac{di}{dt} + Ri = E$. Find i as a function of t. **(06 Marks)**

6 a. Choose the correct answers for the following : **(04 Marks)**

i) If $\lim_{n\to\infty} \frac{u_{n+1}}{u_n} = \lambda$, then the series $\sum u_n$ is convergent by ratio test when _____.

A) λ < 1 B) λ > 1 C) λ = 1 D) λ = 0

ii) The series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent if _____.

A) p < 1 B) p ≤ 1 C) p > 1 D) p < –1

iii) The alternating series $1 - \frac{1}{5} + \frac{1}{9} - \frac{1}{13} + ...\infty$ is

A) convergent B) absolutely convergent
C) conditionally convergent D) oscillatory

iv) The series $1 + r + r^2 + r^3 + ...\ \infty$ is convergent if _____.

A) |r| < 1 B) r ≥ 1 C) r ≥ –1 D) |r| = 1

6 b. Test the convergent of the series $1+\frac{L^2}{2^2}+\frac{L^3}{3^3}+\frac{L^4}{4^4}+......\infty$. **(04 Marks)**

c. Find the nature of the series $\frac{1}{1^2}+\frac{1+2}{1^2+2^2}+\frac{1+2+3}{1^2+2^2+3^2}+.......\infty$. **(06 Marks)**

d. Test for convergence of the series $\frac{4}{3}+\frac{4.7}{3.5}+\frac{4.7.10}{3.5.7}+.........\infty$ **(06 Marks)**

7 a. Choose the correct answers for the following : **(04 Marks)**

i) If (2, 1, 1) and $(4, \sqrt{3}-1, -\sqrt{3}-1)$ be the direction ratios of two lines then angle between the lines is

A) 90° B) 30° C) 45° D) 60°

ii) The normal to a plane make equal angles with the coordinate axis then the direction cosines of the normal to the plane are_____.

A) (1, 1, 1) B) $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

C) $\left(\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$ D) $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}\right)$

iii) The angle between the line $\frac{x-1}{2}=\frac{y-1}{3}=\frac{z-1}{6}$ and the XOY plane is

A) $\cos^{-1}(6/7)$ B) $\sin^{-1}(6/7)$ C) 30° D) 60°

iv) If the shortest distance between two skew lines is zero the lines are

A) parallel B) perpendicular C) coplanar D) non-coplanar

b. Prove that the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$ are coplanar. **(04 Marks)**

c. Find the shortest distance between the lines $\frac{x-3}{1}=\frac{y-5}{-2}=\frac{z-7}{1}$ and $\frac{x+1}{7}=\frac{y+1}{-6}=\frac{z+1}{1}$. Also find its equations. **(06 Marks)**

d. Find the image of the point (1, 3, 4) in the plane $2x - y + z + 3 = 0$. **(06 Marks)**

8 a. Choose the correct answers for the following : **(04 Marks)**

i) If $\overline{A}$ is a constant vector and $\overline{R} = x\hat{i}+y\hat{j}+z\hat{k}$ then $\text{div}(\overline{A}\times\overline{R})$ is _____.

A) O B) $\overline{O}$ C) $\overline{A}$ D) $2\overline{A}$

ii) If $\overline{R} = x\hat{i}+y\hat{j}+z\hat{k}$ then $\nabla\times\overline{R}$ is ____.

A) O B) $3\hat{i}$ C) $\overline{O}$ D) $2\hat{j}$

iii) The magnitude of the acceleration of a particle moving along a curve $x = e^{-t}$; $y = 2\cos 3t$, $z = 2\sin 3t$ at $t = 0$ where t is the time is equal to ______.

A) $\sqrt{37}$ B) $\sqrt{325}$ C) $\sqrt{20}$ D) 0

iv) Any motion in which the curl of the velocity vector is said to be________.

A) rotational B) irrotational C) solenoidal D) scalar

b. If $\overline{F} = (x+y+1)\hat{i}+\hat{j}-(x+y)\hat{k}$, show that $\overline{F}.\text{curl}\overline{F} = 0$. **(04 Marks)**

c. Show that $\nabla(r^n) = nr^{n-1}\hat{r}$ where $\overline{r} = x\hat{i}+y\hat{j}+z\hat{k}$. **(06 Marks)**

d. If $\overline{F} = \nabla(x^3+y^3+z^3-3xyz)$ find i) $\nabla.\overline{F}$, ii) $\nabla\times\overline{F}$. **(06 Marks)**

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